

Answer ALL questions. Write your answers in the spaces provided.

1.

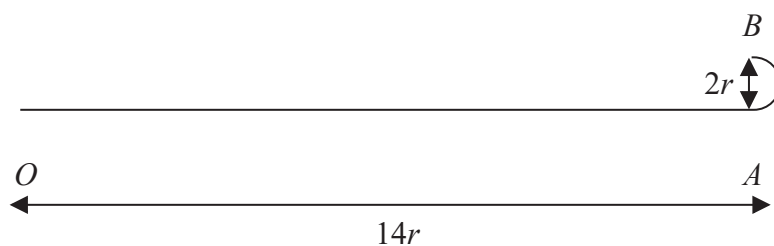


Figure 1

A hockey stick is modelled as a uniform rod OA of length $14r$ joined to a uniform semicircular arc AB of diameter $2r$, as shown in Figure 1. The rod and the arc lie in the same plane and are made of the same material.

- (a) Find, in terms of π and r , the distance of the centre of mass of the hockey stick from the line AB .

(5)

The hockey stick is freely suspended from O and hangs in equilibrium.

Given that the centre of mass of the hockey stick is a distance $\frac{\pi r}{(14 + \pi)}$ from OA ,

- (b) find, in degrees to 3 significant figures, the angle between OA and the vertical.

(2)

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Question 1 continued

Lined area for writing answers.

(Total for Question 1 is 7 marks)



$$\text{is } 10 \ln \left(\frac{3 + 2e}{5} \right) \text{ m.} \quad (4)$$



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Question 2 continued

Lined area for writing answers.

(Total for Question 2 is 9 marks)



- 3.** A thin non-uniform semicircular metal plate has mass M and radius a .

The plate is modelled as a non-uniform semicircular lamina with centre O , radius a and mass M , where the mass per unit area at any point P of the lamina is proportional to the distance OP .

- (a) Show that the constant of proportionality is $\frac{3M}{\pi a^3}$ (5)
- (b) Find, in terms of a , the exact distance of the centre of mass of the plate from O . (4)



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Question 3 continued

Handwriting practice area with horizontal lines.

(Total for Question 3 is 9 marks)



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Question 4 continued

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Question 4 continued

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(Total for Question 4 is 12 marks)



5. A light inextensible string of length a has one end attached to a fixed point O . The other end of the string is attached to a small stone P of mass m . The stone hangs at rest vertically below O , with the string taut, and is then projected horizontally with speed $\sqrt{\frac{22ag}{5}}$. The stone is modelled as a particle.

Using the model,

- (a) show that the speed of P at the instant when the string first goes slack is $\sqrt{\frac{4ag}{5}}$ (7)
- (b) state the magnitude and direction of the acceleration of P at the instant when the string first goes slack, justifying your answers, (2)
- (c) find the speed of P at the instant when it first passes through the horizontal, through O , after the string goes slack. (3)
- (d) Give a reason why your answer to part (c) is unlikely to be the actual speed of the stone at this instant. (1)



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Question 5 continued

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Question 5 continued

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Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 13 marks)



6.

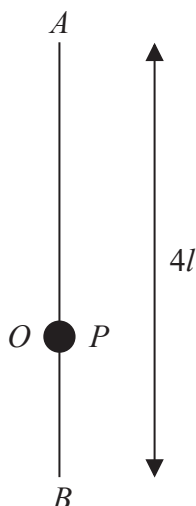


Figure 2

A light elastic string, of natural length l and modulus of elasticity λ , has one end attached to a fixed point A and the other end attached to a particle P of mass m . Another light elastic string, of natural length l and modulus of elasticity 3λ , has one end attached to the particle P and the other end attached to a fixed point B . The point B is vertically below A with $AB = 4l$. The particle P rests in equilibrium, with both strings stretched, at the point O , as shown in Figure 2.

- (a) (i) Show that, if $\lambda > kmg$, where k is a constant

$$AO = \frac{l(10\lambda + mg)}{4\lambda} \quad (4)$$

- (ii) Find the least possible value of k .

(2)

It is now given that $\lambda = 2mg$ and air resistance is modelled as being negligible.

The particle P is projected vertically upwards from O .

Using the model,

- (b) prove that P begins to move with simple harmonic motion of period $2\pi\sqrt{\frac{l}{8g}}$ (6)

- (c) Find, in terms of g and l , the greatest speed with which the particle can be projected vertically upwards from O if P continues to move with simple harmonic motion of period $2\pi\sqrt{\frac{l}{8g}}$ indefinitely. (3)



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Question 6 continued

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Question 6 continued

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Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 15 marks)



7.

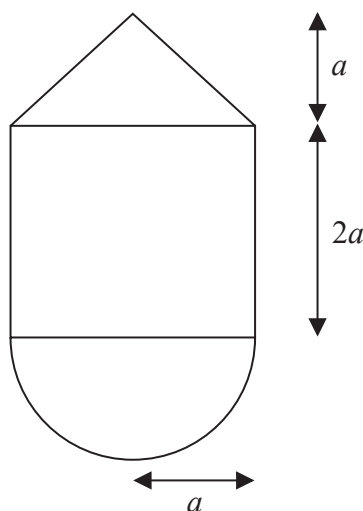


Figure 3

A solid toy consists of a solid hemisphere of radius a , a solid right circular cylinder of radius a and height $2a$, and a solid right circular cone of base radius a and height a , joined together. The plane face of the cone coincides with one of the plane faces of the cylinder and the plane face of the hemisphere coincides with the other plane face of the cylinder. A side elevation of the toy is shown in Figure 3.

The cone and cylinder are made of uniform material of density ρ , and the hemisphere is made of uniform material of density $k\rho$.

The toy is designed so that if it is placed, with its axis vertical and the hemisphere downwards, on a rough horizontal table and then slightly tilted to one side, the toy will return to the vertical position. The table is sufficiently rough to prevent the toy from slipping on the table.

(a) Show, by considering the centre of mass of the toy, that $k > 11$

(7)

If instead $k = 11$ and the toy is placed, with its axis vertical and the hemisphere downwards, on the same rough horizontal table, and then slightly tilted to one side,

(b) describe what happens, explaining your answer.

(3)



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Question 7 continued

Lined area for writing the answer to Question 7.



Question 7 continued

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

